Physics 4A
Chapter 15: Oscillations

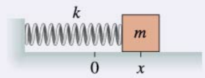
GENERAL PRINCIPLES

Dynamics

SHM occurs when a **linear restoring force** acts to return a system to an equilibrium position.

Horizontal spring

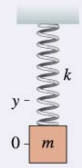
$(F_{net})_x = -kx$



Vertical spring

The origin is at the equilibrium position $\Delta L = mg/k$.

$(F_{net})_y = -ky$

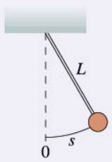


Both: $\omega = \sqrt{\dfrac{k}{m}} \qquad T = 2\pi\sqrt{\dfrac{m}{k}}$

Simple pendulum

$\omega = \sqrt{\dfrac{g}{L}}$

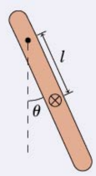
$T = 2\pi\sqrt{\dfrac{L}{g}}$



Physical pendulum

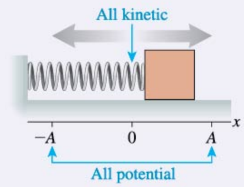
$\omega = \sqrt{\dfrac{Mgl}{I}}$

$T = 2\pi\sqrt{\dfrac{I}{Mgl}}$



Energy

If there is no friction or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy $E = K + U$ is conserved.

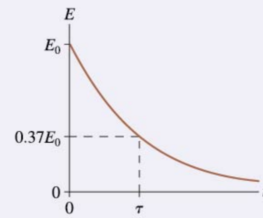


$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$
$$= \tfrac{1}{2}m(v_{max})^2$$
$$= \tfrac{1}{2}kA^2$$

The energy of a lightly damped oscillator decays exponentially

$$E = E_0 e^{-t/\tau}$$

where τ is the **time constant**.



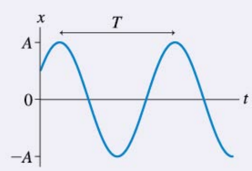
IMPORTANT CONCEPTS

Simple harmonic motion (SHM) is a sinusoidal oscillation with period T and amplitude A.

Frequency $f = \dfrac{1}{T}$

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$



Position $x(t) = A\cos(\omega t + \phi_0)$

$$= A\cos\left(\frac{2\pi t}{T} + \phi_0\right)$$

Velocity $v_x(t) = -v_{max}\sin(\omega t + \phi_0)$ with maximum speed $v_{max} = \omega A$

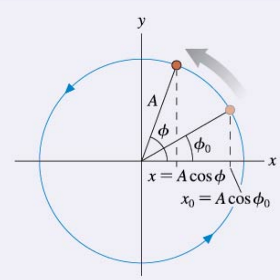
Acceleration $a_x(t) = -\omega^2 x(t) = -\omega^2 A\cos(\omega t + \phi_0)$

SHM is the projection onto the x-axis of **uniform circular motion.**

$\phi = \omega t + \phi_0$ is the **phase**

The position at time t is

$$x(t) = A\cos\phi$$
$$= A\cos(\omega t + \phi_0)$$



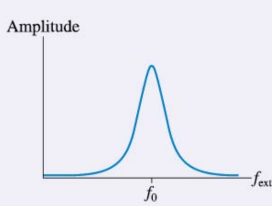
The **phase constant** ϕ_0 is determined by the initial conditions:

$$x_0 = A\cos\phi_0 \qquad v_{0x} = -\omega A\sin\phi_0$$

APPLICATIONS

Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_{ext} \approx f_0$, where f_0 is the system's natural oscillation frequency, or **resonant frequency.**

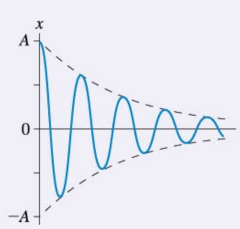


Damping

If there is a drag force $\vec{F}_{drag} = -b\vec{v}$, where b is the damping constant, then (for lightly damped systems)

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

The time constant for energy loss is $\tau = m/b$.



Questions and Example Problems from Chapter 15

Conceptual Question 15.2

A pendulum on Planet X, where the value of g is unknown, oscillates with a period T = 2 s. What is the period of this pendulum if: **(a)** Its mass is doubled? **(b)** Its length is doubled? **(c)** Its oscillation amplitude is doubled?

15.2. The period of a simple pendulum is $T = 2\pi\sqrt{L/g}$. We are told that $T_1 = 2.0$ s.

(a) In this case the mass is doubled: $m_2 = 2m_1$. However, the mass does not appear in the formula for the period of a pendulum; that is, the period does not depend on the mass. Therefore the period is still 2.0 s.

(b) In this case the length is doubled: $L_2 = 2L_1$.

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{L_2/g}}{2\pi\sqrt{L_1/g}} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{2L_1}{L_1}} = \sqrt{2}$$

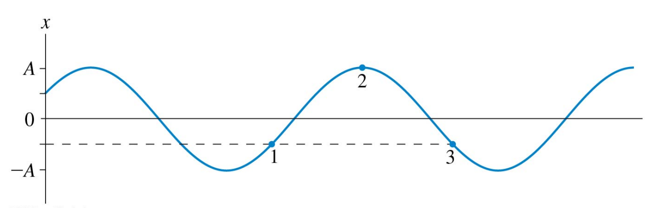
So $T_2 = \sqrt{2}T_1 = \sqrt{2}(2.0\text{ s}) = 2.8$ s.

(c) The formula for the period of a simple small-angle pendulum does not contain the amplitude; that is, the period is independent of the amplitude. Changing (in particular, doubling) the amplitude, as long as it is still small, does not affect the period, so the new period is still 2.0 s.

It is equally important to understand what *doesn't* appear in a formula. It is quite startling, really, the first time you realize it, that the amplitude (θ_{max}) doesn't affect the period. But this is crucial to the idea of simple harmonic motion. Of course, if the pendulum is swung too far, out of its linear region, then the amplitude would matter. The amplitude *does* appear in the formula for a pendulum not restricted to small angles because the small-angle approximation is not valid; but then the motion is not simple harmonic motion.

Conceptual Question 15.4

The figure shows a position-versus-time graph for a particle in SHM. **(a)** What is the phase constant? *Explain.* **(b)** What is the phase of the particle at each of the three numbered points on the graph?

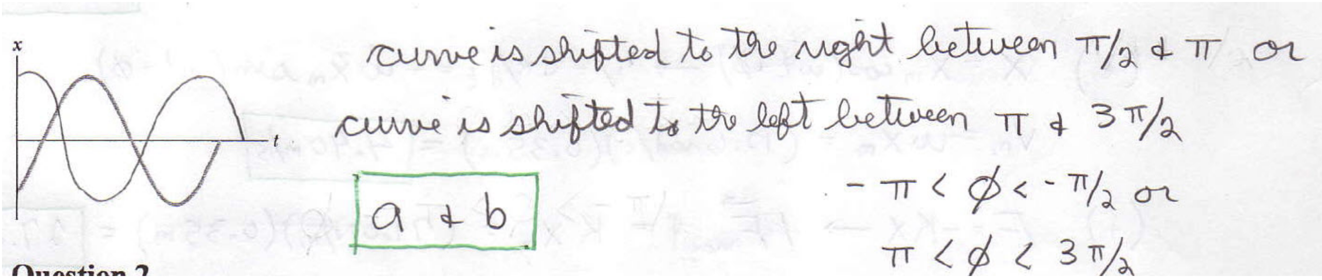


15.4. **(a)** A position-vs-time graph plots $x(t) = A\cos(wt + \phi_0)$. The graph of $x(t)$ starts at $\frac{1}{2}A$ and is increasing. So at $t = 0$, $\frac{1}{2}A = A\cos\phi_0 \Rightarrow \phi_0 = \pm\frac{\pi}{3}$. We choose $\phi_0 = -\frac{\pi}{3}$ since the particle is moving to the right, indicating that it is in the bottom half of the circular-motion diagram.

(b) The phase at each point can be determined in the same manner as for part (a). For points 1 and 3, the amplitude is again $\frac{1}{2}A$. At point 1, the particle is moving to the right so $\phi_1 = -\frac{\pi}{3}$. At point 3, the particle is moving left, so $\phi_3 = +\frac{\pi}{3}$. At point 2, the amplitude is A, so $\cos\phi_2 = 1 \Rightarrow \phi_2 = 0$.

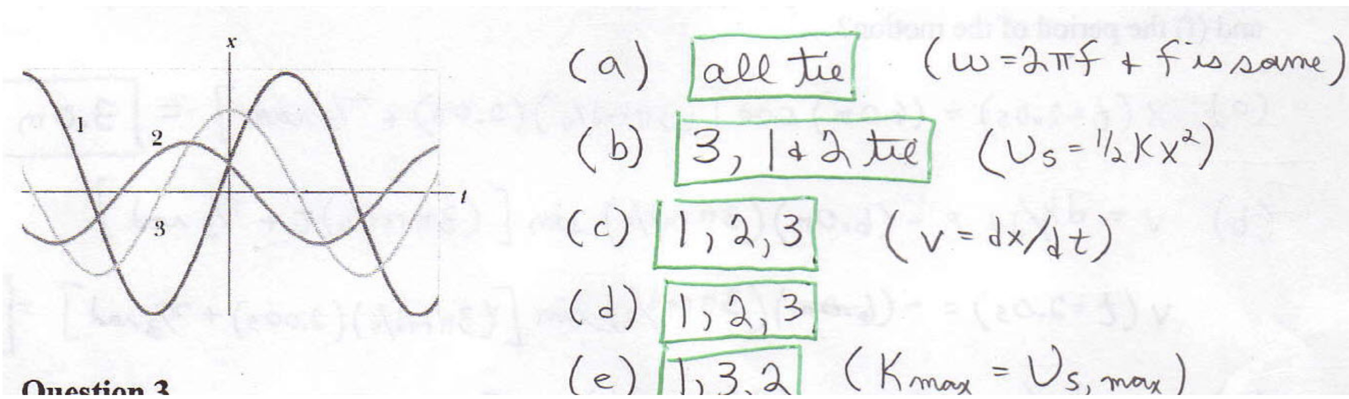
Conceptual Question 15.A

Which of the following describe ϕ for the SHM of the figure below **(a)** $-\pi < \phi < -\pi/2$, **(b)** $\pi < \phi < 3\pi/2$, **(c)** $-3\pi/2 < \phi < -\pi$?



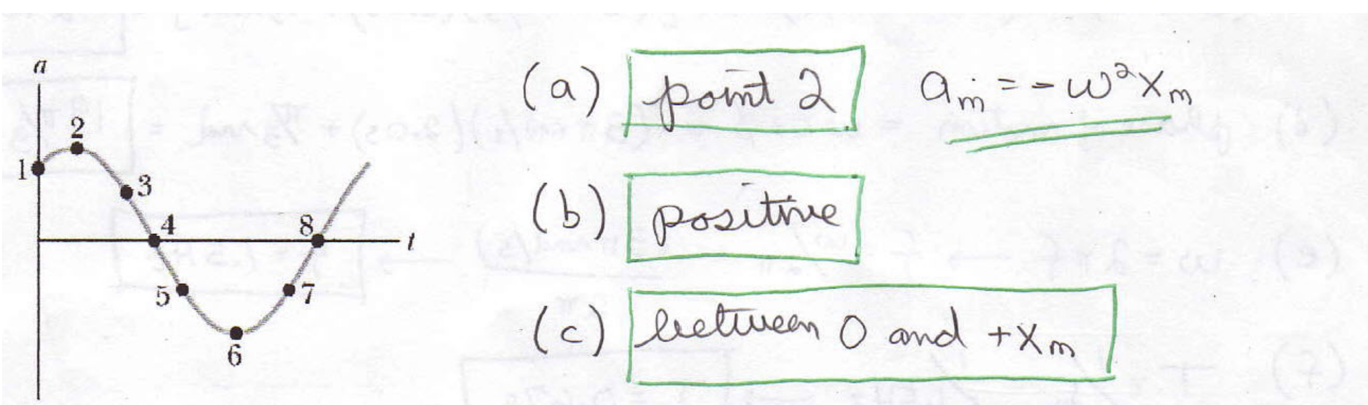
Conceptual Question 15.B

The figure below shows the x(t) curves for three experiments involving a particular spring-mass system oscillating in SHM. Rank the curves according to **(a)** the system's angular frequency; **(b)** the spring's potential energy at time t = 0, **(c)** the box's kinetic energy at t = 0, **(d)** the masses speed at t = 0, and **(e)** the masses maximum kinetic energy, greatest first.



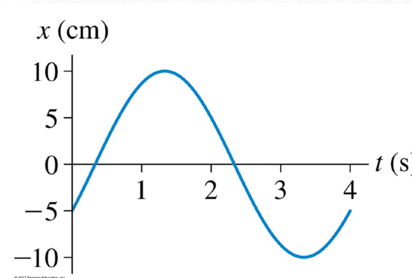
Conceptual Question 15.C

The acceleration a(t) of a particle undergoing SHM is graphed in the figure below. **(a)** Which of the labeled points corresponds to the particle at $-x_m$? **(b)** At point 4, is the velocity of the particle positive, negative, or zero? **(c)** At point 5, is the particle at $-x_m$, at $+x_m$, at 0, between $-x_m$ and 0, or between 0 and $+x_m$?



Problem 15.6

What are the **(a)** amplitude, **(b)** frequency, and **(c)** phase constant of the oscillation shown in the figure to the right?



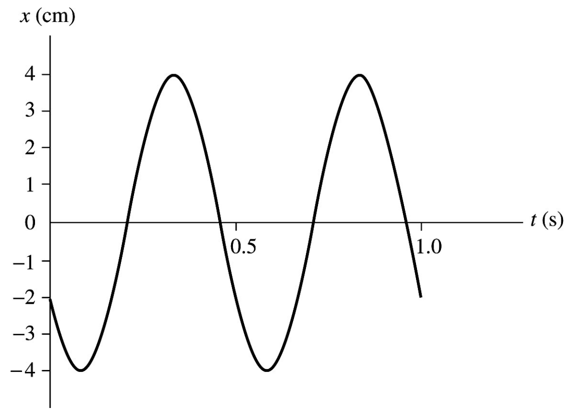
Problem 15.9

An object in simple harmonic motion has an amplitude of 4.0 cm, a frequency of 2.0 Hz, and a phase constant of 2π/3 rad. Draw a position graph showing two cycles of motion.

15.9. Visualize: The phase constant $\frac{2}{3}\pi$ has a plus sign, which implies that the object undergoing simple harmonic motion is in the second quadrant of the circular motion diagram. That is, the object is moving to the left.
Solve: The position of the object is

$$x(t) = A\cos(\omega t + \phi_0) = A\cos(2\pi ft + \phi_0) = (4.0\text{ cm})\cos[(4\pi\text{ rad/s})t + \tfrac{2}{3}\pi\text{ rad}]$$

The amplitude is $A = 4$ cm and the period is $T = 1/f = 0.50$ s. A phase constant $\phi_0 = 2\pi/3\text{ rad} = 120°$ (second quadrant) means that x starts at $-\frac{1}{2}A$ and is moving to the left (getting more negative).



Assess: We can see from the graph that the object starts out moving to the left.

Problem 15.11

An object in simple harmonic motion has amplitude 4.0 cm and frequency 4.0 Hz, and at t = 0 s it passes through the equilibrium point moving to the right. Write the function x(t) that describes the object's position.

15.11. Solve: The position of the object is given by the equation

$$x(t) = A\cos(\omega t + \phi_0) = A\cos(2\pi ft + \phi_0)$$

We can find the phase constant ϕ_0 from the initial condition:

$$0\text{ cm} = (4.0\text{ cm})\cos\phi_0 \Rightarrow \cos\phi_0 = 0 \Rightarrow \phi_0 = \cos^{-1}(0) = \pm\tfrac{1}{2}\pi\text{ rad}$$

Since the object is moving to the right, the object is in the lower half of the circular motion diagram. Hence, $\phi_0 = -\frac{1}{2}\pi$ rad. The final result, with $f = 4.0$ Hz, is

$$x(t) = (4.0\text{ cm})\cos[(8.0\pi\text{ rad/s})t - \tfrac{1}{2}\pi\text{ rad}]$$

Problem 15.16

A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At t = 0 s, the mass is at x = 5.0 cm and has v_x = -30 cm/s. Determine: **(a)** The period. **(b)** The angular frequency. **(c)** The amplitude. **(d)** The phase constant. **(e)** The maximum speed. **(f)** The maximum acceleration. **(g)** The total energy. **(h)** The position at t = 0.40 s.

15.16. Model: The mass attached to the spring oscillates in simple harmonic motion.
Solve: (a) The period $T = 1/f = 1/2.0\text{ Hz} = 0.50$ s.

(b) The angular frequency $\omega = 2\pi f = 2\pi(2.0\text{ Hz}) = 4\pi$ rad/s.

(c) Using energy conservation

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}kx_0^2 + \tfrac{1}{2}mv_{0x}^2$$

Using $x_0 = 5.0$ cm, $v_{0x} = -30$ cm/s and $k = m\omega^2 = (0.200\text{ kg})(4\pi\text{ rad/s})^2$, we get $A = 5.54$ cm.

(d) To calculate the phase constant ϕ_0,

$$A\cos\phi_0 = x_0 = 5.0\text{ cm}$$

$$\Rightarrow \phi_0 = \cos^{-1}\left(\frac{5.0\text{ cm}}{5.54\text{ cm}}\right) = 0.45\text{ rad}$$

(e) The maximum speed is $v_{max} = \omega A = (4\pi\text{ rad/s})(5.54\text{ cm}) = 70\text{ cm/s}$.

(f) The maximum acceleration is

$$a_{max} = \omega^2 A = \omega(\omega A) = (4\pi\text{ rad/s})(70\text{ cm/s}) = 8.8\text{ m/s}^2$$

(g) The total energy is $E = \tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}(0.200\text{ kg})(0.70\text{ m/s})^2 = 0.049$ J.

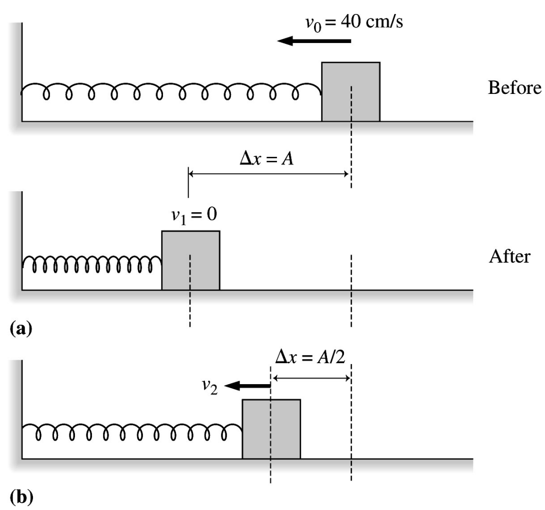
(h) The position at $t = 0.40$ s is

$$x_{0.4\text{ s}} = (5.54\text{ cm})\cos[(4\pi\text{ rad/s})(0.40\text{ s}) + 0.45\text{ rad}] = +3.8\text{ cm}$$

Problem 15.18

A 1.0 kg block is attached to a spring with spring constant 16 N/m. While the block is sitting at rest, a student hits it with a hammer and almost instantaneously gives it a speed of 40 cm/s. What are **(a)** The amplitude of the subsequent oscillations? **(b)** The block's speed at the point where x = ½ A.

15.18. Model: The block attached to the spring is in simple harmonic motion.
Visualize:



Solve: **(a)** The conservation of mechanical energy equation $K_f + U_{sf} = K_i + U_{si}$ is

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}k(\Delta x)^2 = \tfrac{1}{2}mv_0^2 + 0\text{ J} \Rightarrow 0\text{ J} + \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_0^2 + 0\text{ J}$$

$$\Rightarrow A = \sqrt{\frac{m}{k}}v_0 = \sqrt{\frac{1.0\text{ kg}}{16\text{ N/m}}}(0.40\text{ m/s}) = 0.10\text{ m} = 10\text{ cm}$$

(b) We have to find the velocity at a point where $x = A/2$. The conservation of mechanical energy equation $K_2 + U_{s2} = K_i + U_{si}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}mv_0^2 + 0\text{ J} \Rightarrow \frac{1}{2}mv_2^2 = \frac{1}{2}mv_0^2 - \frac{1}{4}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}mv_0^2 - \frac{1}{4}\left(\frac{1}{2}mv_0^2\right) = \frac{3}{4}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow v_2 = \sqrt{\frac{3}{4}}v_0 = \sqrt{\frac{3}{4}}(0.40\text{ m/s}) = 0.346\text{ m/s}$$

The velocity is 35 cm/s.

Problem 15.26

A mass on a string of unknown length oscillates as a pendulum with a period of 4.0 s. What is the period if: **(a)** The mass is doubled? **(b)** The string length is doubled? **(c)** The string length is halved? **(d)** The amplitude is doubled? (Parts a to d are independent questions, each referring to the initial situation.)

15.26. Model: Assume a small angle of oscillation so there is simple harmonic motion.
Solve: The period of the pendulum is

$$T_0 = 2\pi\sqrt{\frac{L_0}{g}} = 4.0 \text{ s}$$

(a) The period is independent of the mass and depends only on the length. Thus $T = T_0 = 4.0$ s.

(b) For a new length $L = 2L_0$,

$$T = 2\pi\sqrt{\frac{2L_0}{g}} = \sqrt{2}T_0 = 5.7 \text{ s}$$

(c) For a new length $L = L_0/2$,

$$T = 2\pi\sqrt{\frac{L_0/2}{g}} = \frac{1}{\sqrt{2}}T_0 = 2.8 \text{ s}$$

(d) The period is independent of the amplitude as long as there is simple harmonic motion. Thus $T = 4.0$ s.

Problem 15.29

Astronauts on the first trip to Mars take along a pendulum that has a period on earth of 1.50 s. The period on Mars turns out to be 2.45 s. What is the free-fall acceleration on Mars?

15.29. Model: Assume a small angle of oscillation so that the pendulum has simple harmonic motion.
Solve: The time periods of the pendulums on the earth and on Mars are

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g_{\text{earth}}}} \quad \text{and } T_{\text{Mars}} = 2\pi\sqrt{\frac{L}{g_{\text{Mars}}}}$$

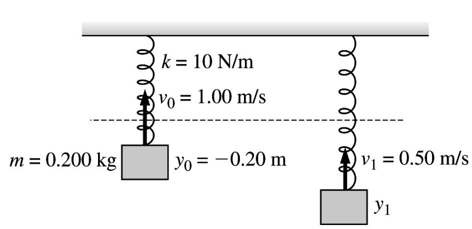
Dividing these two equations,

$$\frac{T_{\text{earth}}}{T_{\text{Mars}}} = \sqrt{\frac{g_{\text{Mars}}}{g_{\text{earth}}}} \Rightarrow g_{\text{Mars}} = g_{\text{earth}}\left(\frac{T_{\text{earth}}}{T_{\text{Mars}}}\right)^2 = (9.8 \text{ m/s}^2)\left(\frac{1.50 \text{ s}}{2.45 \text{ s}}\right)^2 = 3.67 \text{ m/s}^2$$

Problem 15.46

A 200 g block hangs from a spring with spring constant 10 N/m. At t = 0 s the block is 20 cm below the equilibrium point and moving upward with a speed of 100 cm/s. What are the block's **(a)** Oscillation frequency? **(b)** Distance from equilibrium when the speed is 50 cm/s? **(c)** Distance from equilibrium at t = 1.0 s?

15.46. Model: The block undergoes simple harmonic motion.
Visualize:



Solve: (a) The frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{10 \text{ N/m}}{0.20 \text{ kg}}} = 1.125 \text{ Hz}$$

The frequency is 1.1 Hz.

(b) Using conservation of energy, $\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$, we find

$$x_1 = \sqrt{x_0^2 + \frac{m}{k}(v_0^2 - v_1^2)} = \sqrt{(-0.20 \text{ m})^2 + \frac{0.20 \text{ kg}}{10 \text{ N/m}}((1.00 \text{ m/s})^2 - (0.50 \text{ m/s})^2)}$$
$$= 0.2345 \text{ m or } 23 \text{ cm}$$

(c) At time *t*, the displacement is $x = A\cos(\omega t + \phi_0)$. The angular frequency is $\omega = 2\pi f = 7.071$ rad/s. The amplitude is

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-0.20 \text{ m})^2 + \left(\frac{1.00 \text{ m/s}}{7.071 \text{ rad/s}}\right)^2} = 0.245 \text{ m}$$

The phase constant is

$$\phi_0 = \cos^{-1}\left(\frac{x_0}{A}\right) = \cos^{-1}\left(\frac{-0.200 \text{ m}}{0.245 \text{ m}}\right) = \pm 2.526 \text{ rad or } \pm 145°$$

A negative displacement (below the equilibrium point) and positive velocity (upward motion) indicate that the corresponding circular motion is in the third quadrant, so $\phi_0 = -2.526$ rad. Thus at $t = 1.0$ s,

$$x = (0.245 \text{ m})\cos((7.071 \text{ rad/s})(1.0 \text{ s}) - 2.526 \text{ rad}) = -0.0409 \text{ m} = -4.09 \text{ cm}$$

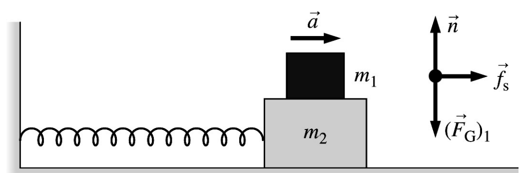
The block is 4.1 cm below the equilibrium point.

Problem 15.52

The two blocks in the figure oscillate on a frictionless surface with a period of 1.5 s. The upper block just begins to slip when the amplitude is increased to 40 cm. What is the coefficient of static friction between the two blocks?

15.52. Model: Assume simple harmonic motion for the two-block system without the upper block slipping. We will also use the model of static friction between the two blocks.

Visualize:



Solve: The net force on the upper block m_1 is the force of static friction due to the lower block m_2. The two blocks ride together as long as the static friction doesn't exceed its maximum possible value. The model of static friction gives the maximum force of static friction as

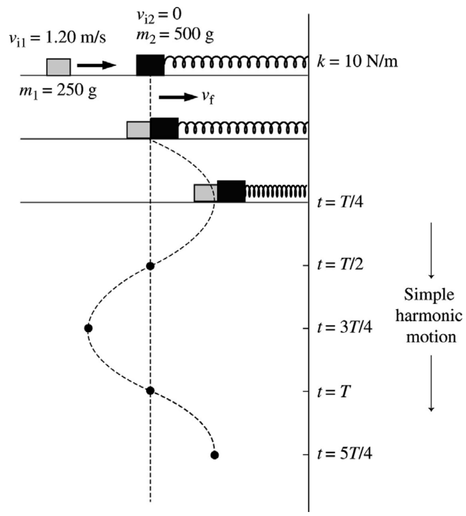
$$(f_s)_{max} = \mu_s n = \mu_s(m_1 g) = m_1 a_{max} \Rightarrow a_{max} = \mu_s g$$
$$\Rightarrow \mu_s = \frac{a_{max}}{g} = \frac{\omega^2 A_{max}}{g} = \left(\frac{2\pi}{T}\right)^2\left(\frac{A_{max}}{g}\right) = \left(\frac{2\pi}{1.5 \text{ s}}\right)^2\left(\frac{0.40 \text{ m}}{9.8 \text{ m/s}^2}\right) = 0.72$$

Assess: Because the period is given, we did not need to use the block masses or the spring constant in our calculation.

Problem 15.60

A 500 g air-track glider attached to a spring with spring constant 10 N/m is sitting at rest on a frictionless air track. A 250 g glider is pushed toward it from the far end of the track at a speed of 120 cm/s. It collides with and sticks to the 500 g glider. What are the amplitude and period of the subsequent oscillations?

15.60. Model: A completely inelastic collision between the two gliders resulting in simple harmonic motion.
Visualize:



Let us denote the 250 g and 500 g masses as m_1 and m_2, which have initial velocities v_{i1} and v_{i2}. After m_1 collides with and sticks to m_2, the two masses move together with velocity v_f.

Solve: The momentum conservation equation $p_f = p_i$ for the completely inelastic collision is $(m_1 + m_2)v_f = m_1 v_{i1} + m_2 v_{i2}$. Substituting the given values,

$$(0.750 \text{ kg})v_f = (0.250 \text{ kg})(1.20 \text{ m/s}) + (0.500 \text{ kg})(0 \text{ m/s}) \Rightarrow v_f = 0.400 \text{ m/s}$$

We now use the conservation of mechanical energy equation:

$$(K + U_s)_{\text{compressed}} = (K + U_s)_{\text{equilibrium}} \Rightarrow 0 \text{ J} + \tfrac{1}{2}kA^2 = \tfrac{1}{2}(m_1 + m_2)v_f^2 + 0 \text{ J}$$

$$\Rightarrow A = \sqrt{\frac{m_1 + m_2}{k}}v_f = \sqrt{\frac{0.750 \text{ kg}}{10 \text{ N/m}}}(0.400 \text{ m/s}) = 0.11 \text{ m}$$

The period is

$$T = 2\pi\sqrt{\frac{m_1 + m_2}{k}} = 2\pi\sqrt{\frac{0.750 \text{ kg}}{10 \text{ N/m}}} = 1.7 \text{ s}$$

Problem 15.A

An oscillator consists of a block of mass 0.500 kg connected to a spring. When set into oscillation with amplitude 35.0 cm, the oscillator repeats its motion every 0.500 s. Find (a) the period, (b) the frequency, (c) the angular frequency, (d) the spring constant, (e) the maximum speed, and (f) the magnitude of the maximum force on the block from the spring.

(a) $T = 0.500\text{ s}$

(b) $f = 1/T = 2.00\text{ Hz}$

(c) $\omega = 2\pi f = 2\pi(2.00\text{ Hz}) \longrightarrow \omega = 12.6\text{ rad/s}$

(d) $\omega = \sqrt{K/m} \longrightarrow K = m\omega^2 = (0.500\text{ kg})(12.6\text{ rad/s}) = 79.0\text{ N/m}$

(e) $x = x_m\cos(\omega t + \phi) \longrightarrow v = dx/dt = -\omega x_m \sin(\omega t + \phi)$

$v_m = \omega x_m = (12.6\text{ rad/s})(0.35\text{ m}) = 4.40\text{ m/s}$

(f) $F = -Kx \longrightarrow |\vec{F}_{max}| = Kx_m = (79.0\text{ N/m})(0.35\text{ m}) = 27.6\text{ N}$

(note: could also use $a_m = \omega^2 x_m$)

Problem 2

Problem 15.B

A body oscillates with simple harmonic motion according to the equation: $x = (6.0\,\text{m})\cos[(3\pi\,\text{rad/s})t + \pi/3\,\text{rad}]$. At t = 2.0 s, what are (a) the displacement, (b) the velocity, (c) the acceleration, and (d) the phase of the motion? Also, what are (e) the frequency and (f) the period of the motion?

(a) $x(t=2.0\text{s}) = (6.0\text{m})\cos[(3\pi\,\text{rad/s})(2.0\text{s}) + \pi/3\,\text{rad}] = \boxed{3.0\text{m}}$

(b) $v = dx/dt = -(6.0\text{m})(3\pi\,\text{rad/s})\sin[(3\pi\,\text{rad/s})t + \pi/3\,\text{rad}]$

$v(t=2.0\text{s}) = -(6.0\text{m})(3\pi\,\text{rad/s})\sin[(3\pi\,\text{rad/s})(2.00\text{s}) + \pi/3\,\text{rad}] = \boxed{-49\,\text{m/s}}$

(c) $a = dv/dt = (-18\pi\,\text{m/s})(3\pi\,\text{rad/s})\cos[(3\pi\,\text{rad/s})t + \pi/3\,\text{rad}]$

$a(t=2.0\text{s}) = (-54\pi^2\,\text{m/s})\cos[(3\pi\,\text{rad/s})(2.0\text{s}) + \pi/3\,\text{rad}] = \boxed{-270\,\text{m/s}^2}$

(d) phase of motion $= \omega t + \phi = (3\pi\,\text{rad/s})(2.0\text{s}) + \pi/3\,\text{rad} = \boxed{19\pi/3\,\text{rad}}$

(e) $\omega = 2\pi f \rightarrow f = \omega/2\pi = \dfrac{(3\pi\,\text{rad/s})}{2\pi} \rightarrow \boxed{f = 1.5\,\text{Hz}}$

(f) $T = 1/f = 1/1.5\text{Hz} \rightarrow \boxed{T = 0.67\text{s}}$

Problem 15.C

A simple harmonic oscillator consists of a block of mass 2.00 kg attached to a spring of spring constant 100 N/m. When t = 1.00 s, the position and velocity of the block are x = 0.129 m and v = 3.415 m/s. (a) What is the amplitude of the oscillations? What were the (b) position and (c) velocity of the block at t = 0 s?

(a) $E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}Kx^2 = \tfrac{1}{2}Kx_m^2 \rightarrow x_m = \sqrt{x^2 + \tfrac{m}{K}v^2}$

$x_m = \sqrt{(0.129\text{m})^2 + \dfrac{(2.00\text{kg})}{100\text{N/m}}(3.415\text{m/s})^2} \rightarrow \boxed{x_m = 0.500\text{m}}$

(b) $x = x_m\cos(\omega t + \phi)$ $\qquad \omega = \sqrt{\dfrac{K}{m}} = 7.07\,\text{rad/s}$ $\quad$ * we need ϕ to get $x + v$ at $t = 0$

$v = -\omega x_m \sin(\omega t + \phi)$

$-v/x = -\omega\tan(\omega t + \phi) \rightarrow \tan(\omega t + \phi) = -v/\omega x \rightarrow \omega t + \phi = \tan^{-1}(-v/\omega x)$

$\phi = \tan^{-1}(-v/\omega x) - \omega t = \tan^{-1}\left[\dfrac{-3.415\text{m/s}}{(7.07\text{rad/s})(0.129\text{m})}\right] - (7.07\text{rad/s})(1.00\text{s}) = -8.38\,\text{rad}$

$x = (0.500\text{m})\cos[(7.07\,\text{rad/s})(0\text{s}) - 8.38\,\text{rad}] = \boxed{-0.251\text{m}}$

Problem 6

(c) $v = -(7.07\text{rad/s})(0.500\text{m})\sin(\omega t + \phi) = \boxed{3.06\text{m/s}}$